FACTORIZATION OF HOLOMORPHIC MATRICES

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Every complex symplectic matrix in $\mathrm{Sp}_{2n}(\mathbb{C})$ can be factorized as a product of the following types of unipotent matrices (in interchanging order).

- (i): $\begin{pmatrix} I & B \\ 0 & I \end{pmatrix}$, upper triangular with symmetric $B = B^T$.
- (ii): $\begin{pmatrix} I & 0 \\ C & I \end{pmatrix}$, lower triangular with symmetric $C = C^T$.

The optimal number $T(\mathbb{C})$ of such factors that any matrix in $\mathrm{Sp}_{2n}(\mathbb{C})$ can be factored into a product of T factors has recently been established to be 5 by Jin, P. Lin, Z. and Xiao, B.

If the matrices depend continuously or holomorphically on a parameter, equivalently their entries are continuous functions on a topological space or holomorphic functions on a Stein space X, it is by no means clear that such a factorization by continuous/holomorphic unipotent matrices exists. A necessary condition for the existence is the map $X \to \mathrm{Sp}_{2n}(\mathbb{C})$ to be null-homotopic. This problem of existence of a factorization is known as the symplectic Vaserstein problem or Gromov-Vaserstein problem. In this talk we report on the results of the speaker and his collaborators B. Ivarsson, E. Low and of his Ph.D. student J. Schott on the complete solution of this problem, establishing uniform bounds $T(d, n)$ for the number of factors depending on the dimension of the space d and the size n of the matrices. It seems difficult to establish the optimal bounds. However we obtain results for the numbers $T(1, n)$, $T(2, n)$ for all sizes of matrices in joint work with our Ph.D. students G. Huang and J. Schott. Finally we give an application to the problem of writing holomorphic symplectic matrices as product of exponentials.

References

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