Port-Hamiltonian Systems and Energy Conversion

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Motivation and context

Many classical and current technological problems are concerned with energy conversion and energy harvesting:

watermills, windmills, steam engine, combustion engines, electrical motors and generators, turbines, fuel cells, etc..

Often multiphysics systems.

Typically, the design and control of such systems is done case by case.

From the heat engine it is known that heat cannot be freely converted into mechanical work: Second law of thermodynamics.

Question:

Is there a theory of energy conversion of general multiphysics systems?

Outline

1. Central question
2. Port-Hamiltonian formulation
3. Structural limitations to energy conversion
4. Energy conversion in case of non-zero off-diagonal blocks
5. Conclusions

Central question of the talk:

Consider a physical system with two power ports

How to convert energy flowing in at port 1 to energy flowing out at port 2, and what are possible limitations in order to do this?

Note: We consider physical systems without internal energy creation, which are thus cyclo-passive :

$$\oint \left[y_1^\top(t) u_1(t) + y_2^\top(t) u_2(t) \right] dt \geq 0$$

for any cyclic motion.

Recall: A function $S(x)$ is a storage function for system with input vector u and equally dimensioned output vector y if

$$S\big(x(t_2)\big) \leq S\big(x(t_1)\big) + \int_{t_1}^{t_2} y^\top(t)u(t)dt$$

holds for all $t_1 \leq t_2$, all input functions $u : [t_1, t_2] \to \mathbb{R}^m$, and all initial conditions $x(t_1)$.

Clearly, if there exists a storage function then the system is cyclo-passive: substitute $x(t_2) = x(t_1)$.

The converse holds under the assumption of reachability from and controllability to some ground state x^*.

The system is passive if $S(x) \geq 0$.

Differential version of the above inequality is

$$\frac{d}{dt}S \leq y^\top u$$

Outline

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2. Port-Hamiltonian formulation
3. Structural limitations to energy conversion
4. Energy conversion in case of non-zero off-diagonal blocks
5. Conclusions

Analysis of energy conversion

Formulate the system into port-Hamiltonian form

$$\begin{aligned}\dot{x} &= J(x)e - \mathcal{R}(x,e) + G(x)u, \quad e = \frac{\partial H}{\partial x}(x),\\ y &= G^\top(x)e, \quad x \in \mathcal{X}\end{aligned}$$

with n-dimensional state space $\mathcal{X}$, Hamiltonian $H : \mathcal{X} \to \mathbb{R}$, skew-symmetric interconnection matrix $J(x) = -J^\top(x)$, and energy-dissipation mapping $\mathcal{R}$ satisfying

$$e^\top \mathcal{R}(x,e) \geq 0, \text{ for all } x, e$$

Any port-Hamiltonian system satisfies

$$\frac{d}{dt}H(x) = e^\top J(x)e - e^\top \mathcal{R}(x,e) + e^\top G(x)u \leq y^\top u$$

Thus H is a storage function, and any port-Hamiltonian system is cyclo-passive.

Port-based modeling of any physical system leads to a port-Hamiltonian formulation.

Port-based modeling is based on representing the system as a network of ideal components, representing

- Energy storage: integrator dynamics defining state variables
- Energy-dissipation: static
- Power-routing elements (ideal transformers, gyrators, $\cdots$): static

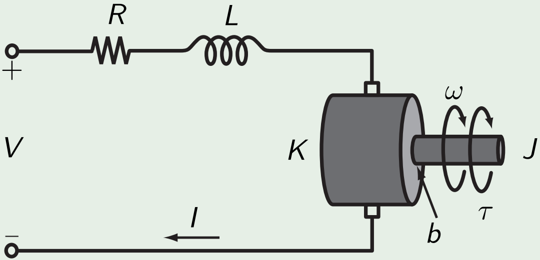
Furthermore, these ideal components are interconnected by pairs of conjugate variables, whose product equal power.

The resulting class of dynamical systems are called port-Hamiltonian systems, although, differently from standard Hamiltonian systems, they do allow for energy dissipation, as well as interaction with the surroundings ('open systems').

Every physical system that is modeled in this way defines a port-Hamiltonian system.

For control purposes 'any' physical system can be modeled this way.

Example (DC motor)



Five interconnected subsystems:

- 2 energy-storing elements: inductor L with state φ (flux), and rotational inertia J with state p (angular momentum);
- 2 energy-dissipating elements: resistor R and friction b;
- gyrator K;

together with electrical port (V, I) and mechanical port (τ, ω).

Example (DC motor cont'd)

The subsystems are interconnected by

$$V_L + V_R + V_K + V = 0, \quad \text{while currents are equal}$$

$$\tau_J + \tau_b + \tau_K + \tau = 0, \quad \text{while angular velocities are equal}$$

Results in port-Hamiltonian model

$$\begin{bmatrix} \dot{\varphi} \\ \dot{p} \end{bmatrix} = \left(\begin{bmatrix} 0 & -K \\ K & 0 \end{bmatrix} - \begin{bmatrix} R & 0 \\ 0 & b \end{bmatrix} \right) \begin{bmatrix} \frac{\varphi}{L} \\ \frac{p}{J} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ \tau \end{bmatrix},$$

$$\begin{bmatrix} I \\ \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\varphi}{L} \\ \frac{p}{J} \end{bmatrix}, \qquad H(\varphi, p) = \frac{\varphi^2}{2L} + \frac{p^2}{2J}$$

Generalization w.r.t. classical Hamiltonian dynamics

$$\begin{aligned} \dot{x} &= J(x)\frac{\partial H}{\partial x}(x) \underline{-\mathcal{R}(x, \frac{\partial H}{\partial x}(x)) + G(x)u} \\ \underline{y} &= \underline{G^T(x)\frac{\partial H}{\partial x}(x)} \end{aligned}$$

Sir William Rowan Hamilton

Addition of

- Energy-dissipating elements
- External ports u, y
- Possibly algebraic constraints
  (not in the above input-state-output formulation)

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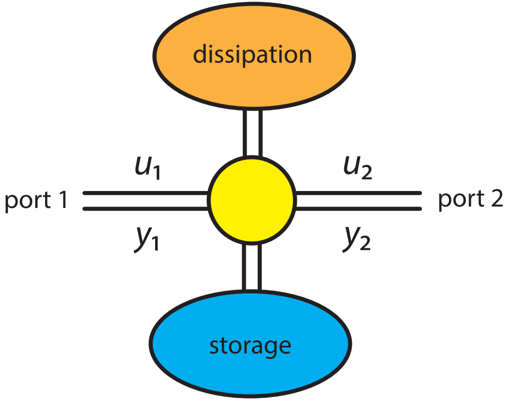
Back to energy conversion

Consider the following cartoon of a port-Hamiltonian system with two ports:



How to convert energy from port 1 to port 2 ?

Hence

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

thus

$$\frac{d}{dt}H \leq y_1^\top u_1 + y_2^\top u_2$$

Energy conversion from port 1 to port 2 if

$$\oint y_1^\top(t)u_1(t)dt \geq 0, \quad \text{energy consumption}$$

while

$$\oint y_2^\top(t)u_2(t)dt \leq 0, \quad \text{energy production}$$

Outline

Suppose there is a partitioning

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

such that the port-Hamiltonian system has block-diagonal structure

$$\begin{aligned}
\dot{x}_1 &= J_1(x_1, x_2)e_1 - \mathcal{R}_1(x_1, x_2, e_1) + G_1(x_1, x_2)u_1, \\
\dot{x}_2 &= J_2(x_1, x_2)e_2 - \mathcal{R}_2(x_1, x_2, e_2) + G_2(x_1, x_2)u_2, \\
y_1 &= G_1^\top(x_1, x_2)e_1, \quad e_1 = \frac{\partial H}{\partial x_1}(x_1, x_2), \\
y_2 &= G_2^\top(x_1, x_2)e_2, \quad e_2 = \frac{\partial H}{\partial x_2}(x_1, x_2),
\end{aligned}$$

where $G_1(x_1, x_2)$ is an invertible matrix for all x_1, x_2. Recall

$$J_1(x) = -J_1^\top(x),\ J_2(x) = -J_2^\top(x), \qquad e_1^\top \mathcal{R}_1(x, e_1) \geq 0,\ e_2^\top \mathcal{R}_2(x, e_2) \geq 0$$

Motions with constant x_1

Hence for all $\bar{x}_1$ there exists input function u_1 that keeps $x_1(t) = \bar{x}_1$.

It follows that for any such input function u_1

$$\frac{d}{dt} H(\bar{x}_1, x_2) = \left[\frac{\partial H}{\partial x_2}(\bar{x}_1, x_2)\right]^\top \dot{x}_2 =$$

$$\left[\frac{\partial H}{\partial x_2}(\bar{x}_1, x_2)\right]^\top \left(J_2(x)e_2 - \mathcal{R}_2(x, e_2) + G_2(x)u_2\right) \leq y_2^\top u_2$$

Hence cyclo-passivity at port 2 with storage function $H(\bar{x}_1, x_2)$: energy is always consumed at port 2 during cyclic motions.

Furthermore, for any t

$$y_1^\top(t)u_1(t) = e_1^\top(t)G_1\big(\bar{x}_1, x_2(t)\big)u_1(t) =$$

$$e_1^\top(t)\big[-J_1\big(\bar{x}_1, x_2(t)\big)e_1(t) + \mathcal{R}_1\big(\bar{x}_1, x_2(t), e_1(t)\big)\big] \geq 0$$

'Static passivity' at port 1.

Summarizing

Theorem

For all cyclic motions with constant $x_1(t) = \bar{x}_1$

$$\oint y_2^\top(t) u_2(t) dt \geq 0$$

Thus system is cyclo-passive at port 2 with storage function $H(\bar{x}_1, x_2)$. Hence no net energy is produced at port 2. Also

$$y_1^\top(t) u_1(t) \geq 0$$

for all t, and thus static passivity at port 1.

Note: inequalities become equalities in case of no dissipation:

$$\mathcal{R}_1(\bar{x}_1, x_2, e_1) = 0, \quad \mathcal{R}_2(\bar{x}_1, x_2, e_2) = 0$$

In analogy with thermodynamics all motions with $x_1(t) = \bar{x}_1$ will be called adiabatics (entropy is constant).

Motions with constant e_1 and y_1

Consider a port-Hamiltonian system with G_1 invertible and constant, and $\frac{\partial^2}{\partial x_1^2} H(x_1, x_2)$ full rank. Since

$$\dot{e}_1 = \frac{\partial^2 H}{\partial x_1^2}(x_1, x_2)\dot{x}_1 + \text{other terms},$$

it follows, after substituting the equation for $\dot{x}_1$, that u_1 can be chosen such that e_1, or equivalently y_1, is constant.

Furthermore, the partial Legendre transform of H with respect to x_1 is

$$H_1^*(e_1, x_2) = H(x_1, x_2) - e_1^\top x_1, \;\; e_1 = \frac{\partial H}{\partial x_1}(x_1, x_2),$$

where x_1 is expressed as a function of e_1, x_2 by means of $e_1 = \frac{\partial H}{\partial x_1}(x_1, x_2)$ (locally guaranteed). Has properties

$$\frac{\partial H_1^*}{\partial e_1}(e_1, x_2) = -x_1, \quad \frac{\partial H_1^*}{\partial x_2}(e_1, x_2) = \frac{\partial H}{\partial x_2}(x_1, x_2)$$

Then for any u_1 such that $y_1 = \bar{y}_1$ and $e_1 = \bar{e}_1$ constant

$$
\begin{aligned}
\frac{d}{dt} H_1^*(\bar{e}_1, x_2) &= -x_1^\top \dot{\bar{e}}_1 + e_2^\top \dot{x}_2 \\
&= e_2^\top J_2(x_1, x_2) e_2 - e_2^\top \mathcal{R}_2(x_1, x_2, e_2) + e_2^\top G_2(x_1, x_2) u_2 \\
&\leq y_2^\top u_2
\end{aligned}
$$

Thus cyclo-passivity at port 2 with storage function $H_1^*(\bar{e}_1, x_2)$.

Furthermore

$$
\int_0^\tau \bar{y}_1^\top u_1(t) dt = \int_0^\tau \bar{e}_1^\top G_1 u_1(t) dt =
$$

$$
\int_0^\tau \bar{e}_1^\top \left[\dot{x}_1(t) + \mathcal{R}_1\big(x_1(t), x_2(t), \bar{e}_1\big) \right] dt \geq \bar{e}_1^\top \big(x_1(\tau) - x_1(0)\big)
$$

Thus cyclo-passivity at port 1 as well, with storage function $\bar{e}_1^\top x_1$.

Summarizing

Theorem

Consider a port-Hamiltonian system with G_1 invertible and constant, and $\frac{\partial^2}{\partial x_1^2} H(x_1, x_2)$ full rank. Then, for all cyclic motions with constant $e_1 = \bar{e}_1$, and thus $y_1 = \bar{y}_1$:

$$\oint \bar{y}_1^\top u_1(t)dt \geq 0, \quad \oint y_2^\top (t) u_2(t)dt \geq 0$$

Hence no net energy is produced at port 2, nor at port 1.

Again, the inequalities become equalities in case

$$\mathcal{R}_1(\bar{x}_1, x_2, e_2) = 0, \quad \mathcal{R}_2(\bar{x}_1, x_2, e_2) = 0$$

Motions for which $e_1 = \bar{e}_1$, and $y_1 = \bar{y}_1$, will be called isothermals.

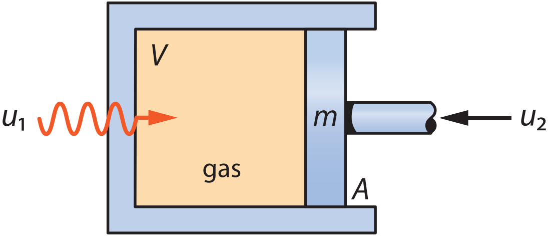


Figure: Gas-piston system with heat port

$$\begin{bmatrix} \dot{S} \\ \dot{V} \\ \dot{\pi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & A \\ 0 & -A & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial S} \\ \frac{\partial H}{\partial V} \\ \frac{\partial H}{\partial \pi} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix},$$

with Hamiltonian $H(S, V, \pi) = U(S, V) + \frac{1}{2m}\pi^2$ and outputs

$$y_1 = \frac{\partial U}{\partial S} \; (= T), \quad y_2 = \frac{\pi}{m}.$$

In case of an ideal gas

$$U(V,S) = \frac{C_V e^{\frac{S}{C_V}}}{V e^{\frac{R}{C_V}}},$$

where C_V denotes the heat capacity (at constant volume) and R is the universal gas constant. The partial Legendre transform with respect to S is the Helmholtz free energy

$$C_V T + W - T\big(C_V \ln T + R \ln V + a\big),$$

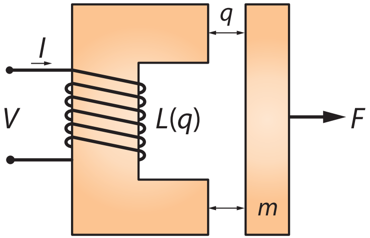
with a the entropy constant, and W an integration constant.

Thus for constant temperature $\bar{T}$ the gas-piston system is cyclo-passive at its mechanical port, with storage function given by

$$C_V \bar{T} + W - \bar{T}\big(C_V \ln \bar{T} + R \ln V + a\big) + \frac{1}{2m}\pi^2 = \bar{T} R \ln V + \frac{1}{2m}\pi^2 + c$$

This is a consequence of the Second Law:

A transformation of a thermodynamic system whose only final result is to transform into work heat extracted from a source which is at the same temperature throughout is impossible.



$$\begin{bmatrix}\dot{\varphi}\\ \dot{q}\\ \dot{p}\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}\begin{bmatrix}\frac{\partial H}{\partial \varphi}\\ \frac{\partial H}{\partial q}\\ \frac{\partial H}{\partial p}\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}V\\ F\end{bmatrix},$$

$$I = \frac{\partial H}{\partial \varphi}, \quad v = \frac{\partial H}{\partial p} \text{ (= velocity of right mass)}$$

Since

$$H(\varphi, q, p) = \frac{\varphi^2}{2L(q)} + \frac{p^2}{2m}$$

the partial Legendre transform wrt to φ is given as

$$H^*_\varphi(I, q, p) = \frac{\varphi^2}{2L(q)} + \frac{p^2}{2m} - \varphi\frac{\varphi}{L(q)}, \quad \varphi = L(q)I$$

and thus

$$H^*_\varphi(I, q, p) = -\frac{1}{2}L(q)I^2 + \frac{p^2}{2m}$$

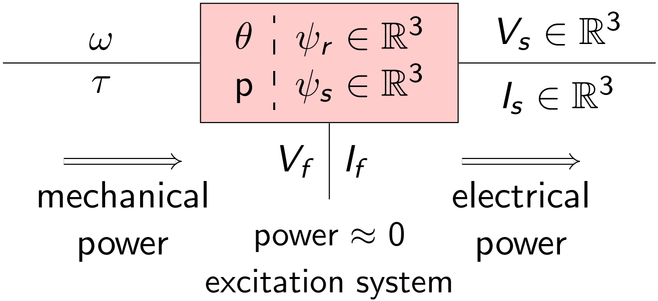
Hence for constant $I = \bar{I}$, the actuator is cyclo-passive at its mechanical port with storage function

$$-\frac{1}{2}L(q)\bar{I}^2 + \frac{p^2}{2m}$$

(NB: a reasonable approximation of the inductance function $L(q)$ is

$$L(q) = \frac{a}{b+q}\ )$$

Synchronous machine/generator



Not cyclo-passive at electrical port for constant angular velocity at mechanical port.

However passive at mechanical port for constant stator currents at electrical port.
('No work produced for constant DC current')

Port-Hamiltonian model

$$\begin{bmatrix} \dot{\psi}_s \\ \dot{\psi}_r \\ \dot{p} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -R_s & 0_{33} & 0_{31} & 0_{31} \\ 0_{33} & -R_r & 0_{31} & 0_{31} \\ 0_{13} & 0_{13} & -b & -1 \\ 0_{13} & 0_{13} & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial \psi_s} \\ \frac{\partial H}{\partial \psi_r} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial \theta} \end{bmatrix} + \begin{bmatrix} I_3 & 0_{31} & 0_{31} \\ 0_{33} & e_1 & 0_{31} \\ 0_{13} & 0 & 1 \\ 0_{13} & 0 & 0 \end{bmatrix} \begin{bmatrix} V_s \\ V_f \\ \tau \end{bmatrix}$$

with outputs

$$\begin{bmatrix} I_s \\ I_f \\ \omega \end{bmatrix} = \begin{bmatrix} I_3 & 0_{33} & 0_{31} & 0_{31} \\ 0_{13} & e_1^\top & 0 & 0 \\ 0_{13} & 0_{13} & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial \psi_s} \\ \frac{\partial H}{\partial \psi_r} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial \theta} \end{bmatrix}$$

with Hamiltonian

$$H(\psi_s, \psi_r, p, \theta) = \frac{1}{2} \begin{bmatrix} \psi_s^\top & \psi_r^\top \end{bmatrix} L^{-1}(\theta) \begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} + \frac{1}{2J_r} p^2,$$

where $J_r > 0$ is rotational inertia and $L(\theta) \succ 0$ is a 6×6 positive-definite symmetric inductance matrix.

For constant stator currents I_s the machine is passive at the combined mechanical and excitation port.

Storage function given by the partial Legendre transform $H^*_{\psi_s}(I_s, \psi_r, p, \theta)$ of $H(\psi_s, \psi_r, p, \theta)$ with respect to ψ_s.

On the other hand, not cyclo-passive at electrical + excitation port for constant velocity ω.

Applying the Blondel-Park transformation the model reduces to a 6-dimensional port-Hamiltonian system in $dq0$ coordinates

$$\begin{bmatrix} \dot{\psi}_d \\ \dot{\psi}_q \\ \dot{\psi}_r \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -\begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix} & 0_{23} & \begin{bmatrix} -\psi_q \\ \psi_d \end{bmatrix} \\ 0_{32} & -R_r & 0_{31} \\ \begin{bmatrix} \psi_q & -\psi_d \end{bmatrix} & 0_{13} & -d \end{bmatrix} \begin{bmatrix} \frac{\partial \widehat{\mathcal{H}}}{\partial \psi_d} \\ \frac{\partial \widehat{\mathcal{H}}}{\partial \psi_q} \\ \frac{\partial \widehat{\mathcal{H}}}{\partial \psi_r} \\ \frac{\partial \widehat{\mathcal{H}}}{\partial p} \end{bmatrix} + \begin{bmatrix} I_2 & 0_{21} & 0_{21} \\ 0_{32} & e_1 & 0_{31} \\ 0_{12} & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{dq} \\ V_f \\ \tau \end{bmatrix}$$

with corresponding outputs.

The Blondel-Park transformation eliminates the dependence of the magnetic energy on the rotor angle θ at the expense of the introduction of non-zero off-diagonal terms in the J-matrix.

The system is not cyclo-passive at the mechanical port anymore. (Constant I_{dq} means there is an alternating current with constant amplitude at the stator side, which does produce a mechanical torque.)

The Carnot cycle for block-diagonal pH systems

Classical solution for energy conversion in this case: the Carnot cycle !

1. On the time-interval $[0, \tau_1]$ consider an isothermal with respect to port 1, corresponding to a constant $e_1 = \bar{e}_1^h$ (h for hot).
2. On the time-interval $[\tau_1, \tau_2]$ consider an adiabatic corresponding to a constant $x_1 = \bar{x}_1$.
3. On the time-interval $[\tau_2, \tau_3]$ consider an isothermal corresponding to a constant $e_1 = \bar{e}_1^c$ (c for cold).
4. Finally, on the time-interval $[\tau_3, \tau_4]$ consider an adiabatic corresponding to a constant $x_1 = \bar{\bar{x}}_1$.

The efficiency of the cycle is the energy delivered via port 2 divided by the supplied energy via port 1 during the first isothermal.

In case $\mathcal{R}_1$ and $\mathcal{R}_2$ are zero, the efficiency is equal to

$$\frac{\bar{e}_1^h \Delta^h x_1 + \bar{e}_1^c \Delta^c x_1}{\bar{e}_1^h \Delta^h x_1} = 1 - \frac{\bar{e}_1^c}{\bar{e}_1^h},$$

where $\Delta^h x_1$ and $\Delta^c x_1$ are the changes in x_1 during the isothermals on $[0, \tau_1]$ and $[\tau_2, \tau_3]$, satisfying

$$\Delta^h x_1 + \Delta^c x_1 = 0$$

Direct generalization of the Carnot efficiency equation.

Outline

Energy conversion is much more easy in case there are off-diagonal blocks in the interconnection matrix J.

Recall the DC-motor

$$\begin{bmatrix} \dot{\varphi} \\ \dot{p} \end{bmatrix} = \left(\begin{bmatrix} 0 & -K \\ K & 0 \end{bmatrix} - \begin{bmatrix} R & 0 \\ 0 & b \end{bmatrix} \right) \begin{bmatrix} \frac{\varphi}{L} \\ \frac{p}{J} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ \tau \end{bmatrix},$$

$$\begin{bmatrix} I \\ \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\varphi}{L} \\ \frac{p}{J} \end{bmatrix}, \qquad H(\varphi, p) = \frac{\varphi^2}{2L} + \frac{p^2}{2J}$$

Thanks to the gyration constant K, energy is easily flowing from the electrical to the mechanical port (or conversely, in case of dynamo mode).

In fact, for constant $I = \bar{I} \neq 0$ the system is not cyclo-passive at the mechanical port; hence net energy can be extracted!

Consider the system with $I = \bar{I} \neq 0$. Then a storage function F for the constrained system should satisfy

$$\frac{d}{dt}F = \frac{dF}{dp}\left[K\frac{\bar{\varphi}}{L} - b\frac{p}{J} + \tau\right] \leq \omega\tau$$

for all τ, where $\frac{\bar{\varphi}}{L} = \bar{I}$.
It follows that $\frac{dF}{dp} = \omega$, and thus that $F(p) = \frac{p^2}{2J} + \text{const}$. After substitution this implies

$$\frac{dF}{dp}\left[K\bar{I} - b\frac{p}{J}\right] = \omega K\bar{I} - b\omega^2 \leq 0$$

for all ω. However, this can only be true whenever $\bar{I} = 0$.

Similar argument holds for the case $\omega = \bar{\omega} \neq 0$ (dynamo mode), showing that the system is not one-port cyclo-passive at the electrical port either, and net energy can be extracted.

Caveat

Consider again the DC motor

$$\begin{bmatrix} \dot{\varphi} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -R & -K \\ K & -b \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial \varphi} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} V + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tau,$$

$$I = \frac{\partial H}{\partial \varphi}$$

$$\omega = \frac{\partial H}{\partial p}$$

with

$$H(\varphi, p) = \frac{\varphi^2}{2L} + \frac{p^2}{2J}$$

Add an integrator $\dot{\theta} = \omega$, to obtain the 3-dimensional port-Hamiltonian model

$$\begin{bmatrix} \dot{\varphi} \\ \dot{p} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -R & -K & 0 \\ K & -b & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial \varphi} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial \theta} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} V + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tau,$$

$$I = \frac{\partial H}{\partial \varphi}$$

$$\omega = \frac{\partial H}{\partial p}$$

where

$$H(\phi, p, \theta) = H(\varphi, p) = \frac{\varphi^2}{2L} + \frac{p^2}{2J}$$

and thus $\frac{\partial H}{\partial \theta} = 0$.

Apply coordinate transformation $\widetilde{\varphi} := \varphi + K\theta$, with p and θ unchanged. This results in the block diagonal pH system

$$\begin{bmatrix} \dot{\widetilde{\varphi}} \\ \dot{p} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -R & 0 & 0 \\ 0 & -b & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \widetilde{H}}{\partial \widetilde{\varphi}} \\ \frac{\partial \widetilde{H}}{\partial p} \\ \frac{\partial \widetilde{H}}{\partial \theta} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} V + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tau,$$

$$I = \frac{\partial \widetilde{H}}{\partial \widetilde{\varphi}}, \quad \omega = \frac{\partial \widetilde{H}}{\partial p}$$

with

$$\widetilde{H}(\widetilde{\varphi}, p, \theta) = \frac{(\widetilde{\varphi} - K\theta)^2}{2L} + \frac{p^2}{2J}$$

Thus for constant $I = \bar{I}$ the extended pH system is cyclo-passive at the mechanical port, with storage function

$$\widetilde{H}^*(\bar{I}, p, \theta) = -\frac{1}{2}L\bar{I}^2 + \frac{p^2}{2J} - K\bar{I}\theta$$

Outline

Conclusions

- Energy conversion in general multiphysics systems can be studied from a port-Hamiltonian perspective.
- Structural limitations to energy conversion in case of block-diagonal pH systems. Similar to thermodynamics.
- Generalization of Carnot cycle to block-diagonal pH systems

◦ Develop strategies different from the Carnot cycle.
E.g., periodic temperature profiles.

◦ Transform the system to a non block-diagonal form by state/input/output transformations (such as Blondel-Parks), or by feedback: leads to matching equations similar to IDA-PBC control.

◦ Is there a suitable notion of optimal energy conversion?

References:

AvdS, D. Jeltsema, "Limits to Energy Conversion", *IEEE TAC*, 2021

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