Dissipative partial differential-algebraic equations

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Finite-dimensional linear DAEs

$$\frac{d}{dt}Ex(t) = Ax(t), \quad t \geq 0, \qquad Ex(0) = z_0.$$

- E, $A \in \mathbb{C}^{n\times n}$, $z_0 \in \mathbb{C}^n$, det $(\lambda E - A) \neq 0$ for some $\lambda \in \mathbb{C}$.

$(E, A) \sim (\tilde{E}, \tilde{A})$, if T, S invertible exist: $TES = \tilde{E}$, $TAS = \tilde{A}$.

Weierstraß canonical form

$(E, A) \sim \left(\begin{bmatrix} I & 0 \\ 0 & N \end{bmatrix}, \begin{bmatrix} J & 0 \\ 0 & I \end{bmatrix} \right)$, where N is nilpotent.

Dissipativity

$\frac{d}{dt}\|Ex(t)\|^2 \leq 0 \iff \mathrm{Re}\,\langle Ex(t), Ax(t)\rangle \leq 0.$

Infinite-dimensional DAEs

$$\frac{d}{dt}Ex(t) = Ax(t), \quad t \geq 0, \qquad Ex(0) = z_0.$$

- $\mathcal{X}$ and $\mathcal{Z}$ are Hilbert spaces
- $E : \mathcal{X} \to \mathcal{Z}$ linear and bounded
- $A : \mathcal{D}(A) \subset \mathcal{X} \to \mathcal{Z}$ densely defined and closed
- $z_0 \in \mathcal{Z}$
- There exists $s \in \mathbb{C}$ such that $sE - A$ is boundedly invertible

Example: Dzektser equation

$$\frac{\partial}{\partial t}\left(1+\frac{\partial^2}{\partial\zeta^2}\right)x(\zeta,t)=\left(\frac{\partial^2}{\partial\zeta^2}+2\frac{\partial^4}{\partial\zeta^4}\right)x(\zeta,t),$$

$t>0$ and $\zeta\in(0,\pi)$, with boundary conditions

$$x(0,t)=x(\pi,t)=0, \quad t>0$$
$$\frac{\partial^2 x}{\partial\zeta^2}(0,t)=\frac{\partial^2 x}{\partial\zeta^2}(\pi,t)=0, \quad t>0.$$

Let $\mathcal{Z}=L^2(0,\pi)$ and $\mathcal{X}=H^2(0,\pi)\cap H_0^1(0,\pi)$ with $\|x\|_{\mathcal{X}}^2=\|x''\|_{\mathcal{Z}}^2$, $E\in\mathcal{L}(\mathcal{X},\mathcal{Z})$ and $A:\mathcal{D}(A)\subset\mathcal{X}\to\mathcal{Z}$ given by

$$\begin{aligned} Ex&=x+x'',\\ Ax&=x''+2x^{(4)},\\ \mathcal{D}(A)&=\{x\in H^4(0,\pi)\cap H_0^1(0,\pi)\mid x''(0)=x''(\pi)=0\}.\end{aligned}$$

What is known?

- Existence of solutions on a subspace: Yagi 1991, Thaller & Thaller 1996/2001, Favini & Yagi 1999/2004, Reis & Tischendorff 2005, Reis 2008, Showalter 2010, Trostorff 2020, ...
- "Weierstraß canonical form":

  - **Thaller & Thaller 1996**: Investigate the splitting $\mathcal{X} = \ker E \oplus \overline{\operatorname{ran} E^*}$ and $\mathcal{Z} = \ker E^* \oplus \overline{\operatorname{ran} E}$.

  - **Sviridyuk & Fedorov 2003**: Characterise solvability in Banach spaces and prove a canonical form.

  - **Reis 2008**: Generalization of the Weierstraß canonical form. Requires the existence of certain projections.

Aim of this talk

Characterize DAEs $\frac{d}{dt}Ex(t) = Ax(t)$ such that

- For $z_0 \in \operatorname{ran} E$, the DAE has a solution $x(\cdot)$: $\frac{d}{dt}\|Ex(t)\|^2 \leq 0$
- $(E, A) \sim \left(\left[\begin{smallmatrix} I & 0 \\ 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} J & 0 \\ 0 & I \end{smallmatrix}\right]\right)$

Infinite-dimensional ODE

$$\dot{x}(t) = Ax(t), \quad t \geq 0, \qquad x(0) = z_0.$$

- $\mathcal{X}$ is a Hilbert spaces, $z_0 \in \mathcal{X}$
- $A : \mathcal{D}(A) \subset \mathcal{X} \to \mathcal{X}$ densely defined and closed
- There exists $s \in \mathbb{C}$ such that $(sI - A)^{-1} \in \mathcal{L}(\mathcal{X})$

We call $x : [0, \infty) \to X$ a classical solution, if x is continuously differentiable and $\dot{x}(t) = Ax(t)$, $t \geq 0$, and $x(0) = z_0$.

We call $x : [0, \infty) \to X$ a mild solution, if x is continuous and $x(t) = z_0 + A \int_0^t x(s)\, ds$ for $t \geq 0$.

Every classical solution is a mild solution.

The following assertions are equivalent:

- for all $z_0 \in \mathcal{X}$ there exists a unique mild solution.
- for all $z_0 \in \mathcal{D}(A)$ there exists a unique classical solution.

Infinite-dimensional ODE

$$\dot{x}(t) = Ax(t), \quad t \geq 0, \qquad x(0) = z_0.$$

$\mathcal{X}$ Hilbert spaces, $z_0 \in \mathcal{X}$, $A : \mathcal{D}(A) \subset \mathcal{X} \to \mathcal{X}$ densely defined & closed

Hille-Yosida Theorem

The following assertions are equivalent:

- for all $z_0 \in \mathcal{X}$ there exists a unique bounded mild solution.
- There is $K \geq 1$: $(0, \infty) \in \varrho(A)$ and $\|(sI - A)^{-n}\|_{\mathcal{L}(\mathcal{X},\mathcal{X})} \leq \frac{K}{s^n}$ for $n \in \mathbb{N}$ and $s > 0$.

Lumer-Phillips Theorem

The following assertions are equivalent:

- for all $z_0 \in \mathcal{X}$ there exists a unique mild solution with non-increasing norm.
- Re $\langle Ax, x \rangle \leq 0$ for $x \in \mathcal{D}(A)$ and ran $(I - A) = \mathcal{X}$.
- Re $\langle Ax, x \rangle \leq 0$ for $x \in \mathcal{D}(A)$ and Re $\langle A^*x, x \rangle \leq 0$ for $x \in \mathcal{D}(A^*)$.

E-radiality (Sviridyuk & Fedorov)

Resolvent set $\varrho(E, A) := \{s \in \mathbb{C} \mid (sE - A)^{-1} \in \mathcal{L}(\mathcal{Z}, \mathcal{X})\}$

$$R^E(s, A) = (sE - A)^{-1}E, \quad L^E(s, A) = E(sE - A)^{-1},$$

The operator A is E-radial, if

- $s \in \varrho(E, A)$ for all real $s > 0$,
- there exists $K > 0$ such that for $n \in \mathbb{N}$ and for $s > 0$

$$\|(R^E(s, A))^n\|_{\mathcal{L}(\mathcal{X},\mathcal{X})} \leq \frac{K}{s^n}, \quad \|(L^E(s, A))^n\|_{\mathcal{L}(\mathcal{Z},\mathcal{Z})} \leq \frac{K}{s^n}$$

Define for some $\alpha \in \varrho(E, A)$,

$$\mathcal{X}^0 = \ker R^E(\alpha, A) = \ker E, \qquad \mathcal{X}^1 = \overline{\operatorname{ran} R^E(\alpha, A)},$$
$$\mathcal{Z}^0 = \ker L^E(\alpha, A), \qquad \mathcal{Z}^1 = \overline{\operatorname{ran} L^E(\alpha, A)}.$$

These spaces are independent of the choice of α.

E-radiality

$$\mathcal{X}^0 = \ker R^E(\alpha, A) = \ker E, \qquad \mathcal{X}^1 = \overline{\operatorname{ran} R^E(\alpha, A)},$$
$$\mathcal{Z}^0 = \ker L^E(\alpha, A), \qquad \mathcal{Z}^1 = \overline{\operatorname{ran} L^E(\alpha, A)}.$$

If, A is E-radial, then we have:

- $\mathcal{X} = \mathcal{X}^0 \oplus \mathcal{X}^1$ and $\mathcal{Z} = \mathcal{Z}^0 \oplus \mathcal{Z}^1$.
- $P : \mathcal{X} \to \mathcal{X}$ defined by $Px := \lim_{s\to\infty} sR^E(s, A)x$ is a projection with
$$\ker P = \mathcal{X}^0 \text{ and } \operatorname{ran} P = \mathcal{X}^1,$$
- $Q : \mathcal{Z} \to \mathcal{Z}$ defined by $Qz := \lim_{s\to\infty} sL^E(s, A)z$ is a projection with
$$\ker Q = \mathcal{Z}^0 \text{ and } \operatorname{ran} Q = \mathcal{Z}^1,$$
- for all $x \in D(A)$, $Px \in D(A)$ and $APx = QAx$,
- for all $x \in \mathcal{X}$, $EPx = QEx$.

E-radiality

Let A be E-radial. Then the operators

$$\tilde{P} = \begin{bmatrix} I - P \\ P \end{bmatrix} \in \mathcal{L}(\mathcal{X}, \mathcal{X}^0 \times \mathcal{X}^1), \quad \tilde{Q} = \begin{bmatrix} I - Q \\ Q \end{bmatrix} \in \mathcal{L}(\mathcal{Z}, \mathcal{Z}^0 \times \mathcal{Z}^1),$$

are bounded invertible.

$$\begin{aligned} \frac{d}{dt} Ez = Az \quad &\iff \quad \frac{d}{dt} \tilde{Q} E \tilde{P}^{-1} \begin{bmatrix} z_0 \\ z_1 \end{bmatrix} = \tilde{Q} A \tilde{P}^{-1} \begin{bmatrix} z_0 \\ z_1 \end{bmatrix} \\ &\iff \quad \frac{d}{dt} \begin{bmatrix} E_0 & 0 \\ 0 & E_1 \end{bmatrix} \begin{bmatrix} z_0 \\ z_1 \end{bmatrix} = \begin{bmatrix} A_0 & 0 \\ 0 & A_1 \end{bmatrix} \begin{bmatrix} z_0 \\ z_1 \end{bmatrix} \end{aligned}$$

If additionally $\operatorname{ran} E$ is closed, then we have:

- $E_0 \in \mathcal{L}(\mathcal{X}^0, \mathcal{Z}^0)$ with $E_0 = 0$,
- $E_1 \in \mathcal{L}(\mathcal{X}^1, \mathcal{Z}^1)$ is boundedly invertible.
- $A_0 : D(A_0) \subset \mathcal{X}^0 \to \mathcal{Z}^0$ is densely defined, closed & boundedly invertible,
- $A_1 : D(A_1) \subset \mathcal{X}^1 \to \mathcal{Z}^1$ is densely defined and closed.

E-radiality

$$\frac{d}{dt}Ex(t) = Ax(t), \quad t \geq 0, \qquad Ex(0) = z_0.$$

- $\mathcal{X}$ and $\mathcal{Z}$ are Hilbert spaces, $z_0 \in \mathcal{Z}$
- $E : \mathcal{X} \to \mathcal{Z}$ linear, bounded with closed range
- $A : \mathcal{D}(A) \subset \mathcal{X} \to \mathcal{Z}$ densely defined and closed
- (E-radial) there exists $K > 0$ such that for $n \in \mathbb{N}$ and $s > 0$ $\|(R^E(s,A))^n\|_{\mathcal{L}(\mathcal{X},\mathcal{X})} \leq \frac{K}{s^n}, \quad \|(L^E(s,A))^n\|_{\mathcal{L}(\mathcal{Z},\mathcal{Z})} \leq \frac{K}{s^n}$

Theorem

There exists invertible operators $T \in L(\mathcal{Z},\mathcal{Z})$ and $S \in L(\mathcal{X},\mathcal{X})$:

$$TES = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \qquad TAS = \begin{bmatrix} I & 0 \\ 0 & A_1E_1^{-1} \end{bmatrix}.$$

Moreover, the reduced system $\dot{z}_1(t) = A_1E_1^{-1}z_1(t)$, $t \geq 0$, $z_1(0) = z$, has a unique mild solution for every $z \in \mathcal{Z}_1$.
If $K = 1$, then the solutions are non-increasing in norm.

Dissipative DAE

$$\frac{d}{dt}Ex(t) = Ax(t), \quad t \geq 0, \qquad Ex(0) = z_0.$$

We have (formally) for classical solutions

$$\frac{d}{dt}\|Ex(t)\|^2 = 2\mathrm{Re}\,\langle Ex(t), Ax(t)\rangle$$

and for $s > 0$ and $x \in \mathcal{D}(A)$

$$\begin{aligned} \|(sE - A)x\|_{\mathcal{Z}} &\geq s\|Ex\|_{\mathcal{Z}} && \iff \\ s^2\|Ex\|^2_{\mathcal{Z}} - 2s\mathrm{Re}\,\langle Ex, Ax\rangle + \|Ax\|^2_{\mathcal{Z}} &\geq s^2\|Ex\|^2_{\mathcal{Z}} && \iff \\ \mathrm{Re}\,\langle Ex, Ax\rangle &\leq \frac{1}{s}\|Ax\|^2_{\mathcal{Z}} && . \end{aligned}$$

Let $(0, \infty) \in \varrho(E, A)$. The following are equivalent

- $\mathrm{Re}\,\langle Ex, Ax\rangle \leq 0$ for $x \in \mathcal{D}(A)$.
- $\|(sE - A)x\|_{\mathcal{Z}} \geq s\|Ex\|_{\mathcal{Z}}$ for every $s > 0$, $x \in \mathcal{D}(A)$.
- $\|E(sE - A)^{-1}\| \leq \frac{1}{s}$ for every $s > 0$.

Dissipative DAE

$$\frac{d}{dt}Ex(t) = Ax(t), \quad t \geq 0, \qquad Ex(0) = z_0.$$

- $\mathcal{X}$ and $\mathcal{Z}$ are Hilbert spaces, $z_0 \in \mathcal{Z}$
- $E : \mathcal{X} \to \mathcal{Z}$ linear, bounded with closed range
- $A : \mathcal{D}(A) \subset \mathcal{X} \to \mathcal{Z}$ densely defined and closed
- $\lambda \in \varrho(E, A)$ for some $\lambda > 0$ and

$$\begin{aligned} \operatorname{Re} \langle Ax, Ex \rangle_{\mathcal{Z}} \leq 0, &\quad x \in \mathcal{D}(A), \\ \operatorname{Re} \langle A^*x, E^*x \rangle_{\mathcal{X}} \leq 0, &\quad x \in \mathcal{D}(A^*), \end{aligned}$$

Theorem

There exists invertible operators $T \in L(\mathcal{Z}, \mathcal{Z})$ and $S \in L(\mathcal{X}, \mathcal{X})$:

$$TES = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \qquad TAS = \begin{bmatrix} I & 0 \\ 0 & A_1 E_1^{-1} \end{bmatrix}.$$

Moreover, the reduced system $\dot{z}_1(t) = A_1 E_1^{-1} z_1(t)$, $t \geq 0$, $z_1(0) = z$, has for every $z \in \mathcal{Z}_1$ unique mild solution.
Further, every classical solution of the DAE satisfies $\frac{d}{dt}\|Ex(t)\|^2 \leq 0$.

Can this be generalized to higher nilpotency degree?

So far: $N = 0$, that is, the nilpotency index is 0 or 1.

Approach can be generalized

p-E-radial ($p \in \mathbb{N}$) instead of E-radial: There exists $K > 0$ such that for $n \in \mathbb{N}$ and $s_1, \ldots, s_p > 0$

$$\left\| \left(\prod_{q=0}^{p} R^E(s_q, A) \right)^n \right\|_{\mathcal{L}(\mathcal{X},\mathcal{X})} \leq K \prod_{q=0}^{p} \frac{1}{s_q^n},$$

$$\left\| \left(\prod_{q=0}^{p} L^E(s_q, A) \right)^n \right\|_{\mathcal{L}(\mathcal{Z},\mathcal{Z})} \leq K \prod_{q=0}^{p} \frac{1}{s_q^n}$$

Then

$$TES^{-1} = \begin{bmatrix} N & 0 \\ 0 & I \end{bmatrix},$$

Example: Dzektser equation

$$\frac{\partial}{\partial t}\left(1+\frac{\partial^2}{\partial \zeta^2}\right)x(\zeta,t)=\left(\frac{\partial^2}{\partial \zeta^2}+2\frac{\partial^4}{\partial \zeta^4}\right)x(\zeta,t),\quad t>0,\zeta\in(0,\pi)$$
$$x(0,t)=x(\pi,t)=0,\quad t>0$$
$$\frac{\partial^2 x}{\partial \zeta^2}(0,t)=\frac{\partial^2 x}{\partial \zeta^2}(\pi,t)=0,\quad t>0.$$

Let $\mathcal{Z}=L^2(0,\pi)$ and $\mathcal{X}=H^2(0,\pi)\cap H_0^1(0,\pi)$ with $\|x\|_{\mathcal{X}}^2=\|x''\|_{\mathcal{Z}}^2$,

$$\begin{aligned}
Ex&=x+x'',\\
Ax&=x''+2x^{(4)},\\
\mathcal{D}(A)&=\{x\in H^4(0,\pi)\cap H_0^1(0,\pi)\mid x''(0)=x''(\pi)=0\}.
\end{aligned}$$

For $x\in\mathcal{D}(A)$ we calculate

$$\begin{aligned}
\operatorname{Re}\langle Ax,Ex\rangle_{\mathcal{Z}} &= \operatorname{Re}\int_0^\pi (x''+2x^{(4)})(\overline{x}+\overline{x}'')d\zeta\\
&=-\|x'\|_{L^2(0,\pi)}^2+\|x''\|_{L^2(0,\pi)}^2-2\|x^{(3)}\|_{L^2(0,\pi)}^2-2\operatorname{Re}\int_0^\pi x^{(3)}\overline{x}'\,d\zeta\\
&\le \|x''\|_{L^2(0,\pi)}^2-\|x^{(3)}\|_{L^2(0,\pi)}^2\\
&\le 0.
\end{aligned}$$

Example: Dzektser equation

It Is easy to see that $1 \in \varrho(E, A)$. Next we calculate $A^* : \mathcal{D}(A^*) \subset \mathcal{Z} \to \mathcal{X}$. Note that $S : \mathcal{X} \to \mathcal{Z}$ given by $Sf := f''$ is an isometric isomorphism with

$$(S^{-1}f)(x) = \int_0^x (x-t)f(t)dt - \frac{x}{\pi}\int_0^\pi (\pi - t)f(t)dt.$$

Then $A^*z = S^{-1}z + 2z$ for $z \in \mathcal{X}$. For $x \in \mathcal{D}(A^*) = \mathcal{X}$ and $y = S^{-1}x$ we calculate

$$\begin{aligned}
\operatorname{Re}\langle A^*x, E^*x\rangle_{\mathcal{X}} &= \operatorname{Re}\langle EA^*x, x\rangle_{\mathcal{Z}} \\
&= \operatorname{Re}\int_0^\pi (S^{-1}x + x + 2x + 2x'')\overline{x}d\zeta \\
&= \operatorname{Re}\int_0^\pi (y + y'' + 2y'' + 2y^{(4)})\overline{y''}d\zeta \\
&= -\|y'\|_{\mathcal{Z}}^2 + \|y''\|_{\mathcal{Z}}^2 \\
&\quad - 2\operatorname{Re}\int_0^\pi y'\overline{y^{(3)}}d\zeta - 2\|y^{(3)}\|_{\mathcal{Z}}^2 \\
&= \|y''\|_{\mathcal{Z}}^2 - \|y^{(3)}\|_{\mathcal{Z}}^2 \\
&\leq 0.
\end{aligned}$$

Coupled systems

$$\frac{d}{dt}(\underbrace{\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}}_{E} x(t)) = \underbrace{\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}}_{A} x(t), \qquad t > 0,$$

$A_i : \mathcal{D}(A_i) \subset Z \to Z$ closed & densely defined, $\mathcal{X} = \mathcal{Z} = Z \times Z$, $\mathcal{D}(A) = (\mathcal{D}(A_1) \cap \mathcal{D}(A_3)) \times (\mathcal{D}(A_2) \cap \mathcal{D}(A_4))$.

- Let $0 \in \varrho(A_4)$, $\mathcal{D}(A_4) \subset \mathcal{D}(A_2)$ and $\mathcal{D}(A_4^*) \subset \mathcal{D}(A_3^*)$ and $\overline{A_2 A_4^{-1} A_3} \in \mathcal{L}(Z)$.
- Let there exist $M \geq 1$ and $\omega \in \mathbb{R}$ such that for every $s > \omega$, $s \in \varrho(A_1)$ and $\|(s - A_1)^{-n}\| \leq \frac{M}{(s-\omega)^n}, \quad s > \omega, n \in \mathbb{N}$.

Then $\overline{A} - \omega_0 E$ is E-radial and $\operatorname{ran} E$ is closed.
The projections P and Q for this class of systems are given by

$$P \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{I}{-A_4^{-1} A_3} & 0 \\ & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

and

$$Q \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} I & -A_2 A_4^{-1} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Finite-dimensional linear Hamiltonian DAEs

$$\frac{d}{dt}Ex(t) = AQx(t), \quad t \geq 0, \qquad Ex(0) = z_0.$$
$$H(x) = \langle x, E^*Qx \rangle$$

- E, Q, $A \in \mathbb{C}^{n \times n}$, $z_0 \in \mathbb{C}^n$, det $(\lambda E - AQ) \neq 0$ for some $\lambda \in \mathbb{C}$, $E^*Q = Q^*E \geq 0$.

Dissipativity

$\frac{d}{dt}H(x) \leq 0 \iff A = J - R$ with $J^* = -J$ and $R^* = R \geq 0$.

Dissipative Hamiltonian DAE

$$\frac{d}{dt}Ex(t) = AQx(t), \quad t \geq 0, \qquad Ex(0) = z_0.$$
$$H(x) = \langle x, E^*Qx \rangle$$

- $E \in L(\mathcal{X}, \mathcal{Z})$ closed range, $Q \in L(\mathcal{X}, \mathcal{Z})$ invertible with $E^*Q = Q^*E \geq 0$
- $A : \mathcal{D}(A) \subset \mathcal{Z} \to \mathcal{Z}$ closed & $\mathrm{Re}\,\langle Ax, x \rangle \leq 0, x \in \mathcal{D}(A)$
- There exists $s \in \mathbb{C}$ such that $sE - AQ$ is boundedly invertible

Then there is $X \in L(\mathcal{Z}, \mathcal{Z})$ invertible, $X > 0$:
$$E^*XE = E^*Q \text{ \& } H(x) = \langle Ex, XEx \rangle.$$

Theorem

Suppose $\mathrm{Re}\,\langle A^*x, x \rangle \leq 0$, $x \in \mathcal{D}(A^*)$. *Then there exists invertible operators* $T \in L(\mathcal{Z}, \mathcal{Z})$ *and* $S \in L(\mathcal{X}, \mathcal{X})$*:*

$$TES = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \qquad TAQS = \begin{bmatrix} I & 0 \\ 0 & A_1E_1^{-1} \end{bmatrix}.$$

The reduced system $\dot{z}_1(t) = A_1E_1^{-1}z_1(t)$, *has unique mild solutions. Further, every classical solution of the DAE satisfies* $\frac{d}{dt}H(x(t)) \leq 0$.

Conclusions and future work

We characterized DAEs $\frac{d}{dt}Ex(t) = Ax(t)$ such that

- For $x(0) \in \operatorname{ran} E$, the DAE has a solution $x(\cdot)$: $\frac{d}{dt}\|Ex(t)\|^2 \leq 0$
- $(E, A) \sim \left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} J & 0 \\ 0 & A_1 \end{bmatrix} \right)$
- We generalized Lumer-Phillips Theorem for infinite DAEs.

Future Work:

- port-Hamiltonian DAEs in infinite-dimensional systems
- port-Hamiltonian boundary control DAEs

B.J. and Kirsten Morris: On solvability of dissipative partial differential-algebraic equations, appears in : IEEE Control Systems Letters, 2022

Thanks for your attention!