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A Compositional port-Hamiltonian Approach of Distributed Neural Network Optimal Control with Stability Guarantees

Preprint available

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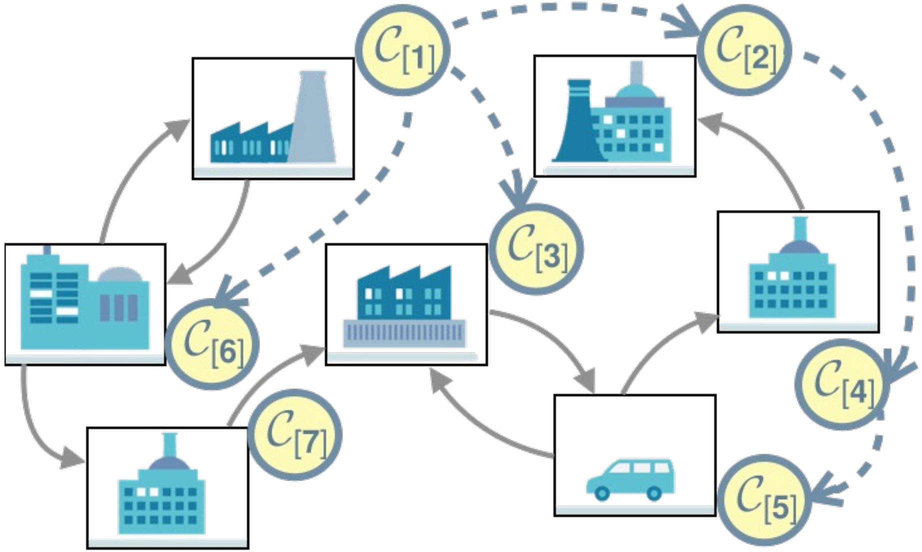
Workshop “Trends on Dissipativity in Systems and Control”, 24.05.2022

Optimal distributed control

- Large scale systems + Optimal distributed control



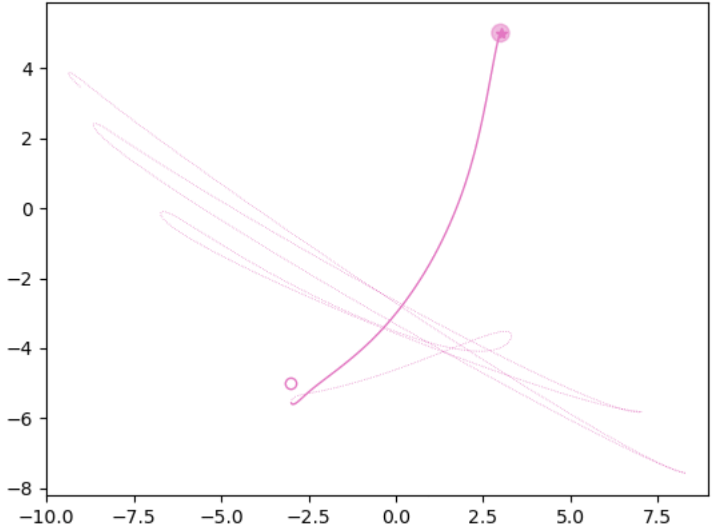
- **Challenge:** non-linear control policies are required
  - Linear systems and quadratic cost (Witsenhausen counterexample)
  - Non-linear systems and/or non-quadratic cost

Deep Neural Networks (DNNs) for parametrizing non-linear policies

Clara Galimberti

Challenges of using DNN policies

- Closed-loop stability guarantees while optimizing transient performance?
  - Not with general multilayer perceptron networks



- Performance optimization over *[0,T]*

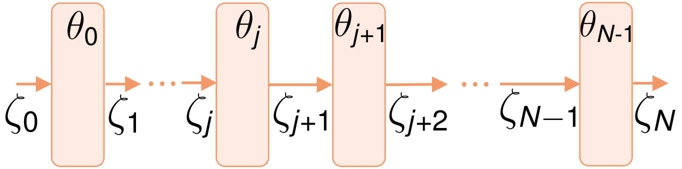
$$\min \int_0^T ||x - x^\star||^2 + ||u||^2 dt$$

s.t. MLP-controller

- Stability if controller applied for $t > T$?

- Stability if optimization stops prematurely?

Challenges of using DNN policies

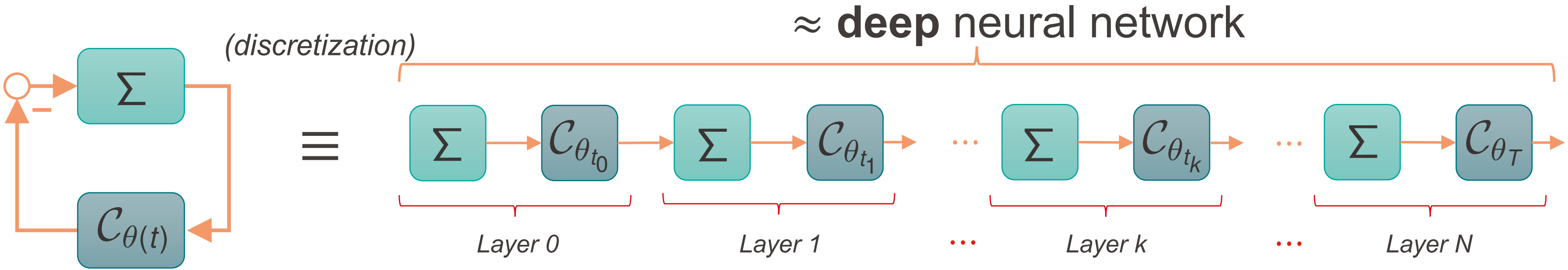


- Vanishing gradients during optimization
  - Backpropagation → Gradient descent

$$\frac{\partial \mathcal{L}}{\partial \theta_{i,j}} = \frac{\partial \zeta_{j+1}}{\partial \theta_{i,j}} \underbrace{\prod_{\ell=j+1}^{N-1} \frac{\partial \zeta_{\ell+1}}{\partial \zeta_{\ell}}}_{\text{Backward sensitivity matrix (BSM)}} \frac{\partial \mathcal{L}}{\partial \zeta_N}$$

  - If $\frac{\partial \mathcal{L}}{\partial \theta_{i,j}} \approx 0 \rightarrow$ Local minima? ☺ / BSM small? ☹

- Optimization for a long control horizon $\equiv$ optimization of a DNN



Challenges of using DNN policies

- Distributed NN control architectures

Graph Neural Networks for Distributed Linear-Quadratic Control

Fernando Gama* FGAMA@BERKELEY.EDU and **Somayeh Sojoudi** SOJOUDI@BERKELEY.EDU
Electrical Engineering and Computer Sciences Dept., University of California, Berkeley, CA 94709, USA

The linear-quadratic co
solution is a linear cont

Communication Topology Co-Design in Graph Recurrent Neural Network based Distributed Control

Fengjun Yang† and Nikolai Matni*

Abstract—**When designing large-scale distributed con-** enjoy approximation guarantees, see for example [1]–[4

  - *A posteriori* analysis of the closed-loop stability

- **Question:** Distributed NN controllers guaranteeing closed-loop stability *by design*?

Our contributions

For port-Hamiltonian systems:

- NN model-based controllers guaranteeing closed-loop stability
  - Optimization of an arbitrary cost over a finite horizon

$$\min_{\theta(t)} \quad \int_0^T \ell(\mathbf{x}(t), \mathbf{u}(t), \theta(t))\, dt$$
$$\text{s.t.} \quad \text{closed-loop stability}$$

- Additional requirement:
  Non-vanishing gradients during optimization

- … even in a distributed setting!

Outline

- Optimal control problem with pH-NN controllers
  - Port Hamiltonian systems
  - pH-NN controller architecture
- Distributed implementations of pH-NN controllers
- Numerical validations
  - pH-NN controllers for robots navigation task
- Conclusions

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- **Optimal control problem with pH-NN controllers**
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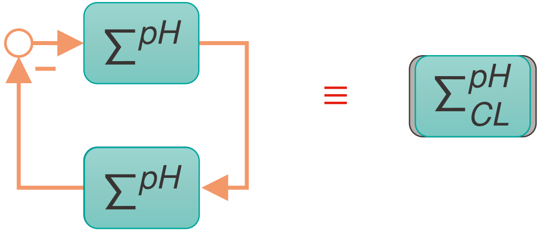
Port-Hamiltonian (pH) systems[1]

$$\dot{\mathbf{x}}(t) = (\boldsymbol{\Omega} - \mathbf{R})\frac{\partial V(\mathbf{x}(t))}{\partial \mathbf{x}} + \mathbf{G}^{\top}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{G}\frac{\partial V(\mathbf{x}(t))}{\partial \mathbf{x}}$$

- $\boldsymbol{\Omega}$ skew-symmetric
- $\mathbf{R} \succeq 0$

- V: Hamiltonian function
  - Continuously differentiable
  - Radially unbounded
- Attractivity: $\lim_{t\to\infty} \mathbf{x}(t) \in \left\{ \mathbf{R}\frac{\partial V}{\partial \mathbf{x}} = 0 \right\}$
- Compositionality:



[1] A. van der Schaft and D. Jeltsema. "Port-Hamiltonian systems theory: An introductory overview." *Foundations and Trends in Systems and Control* 1.2-3 (2014): 173-378.

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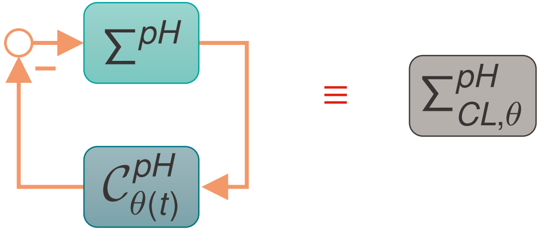


[1] A. van der Schaft and D. Jeltsema. "Port-Hamiltonian systems theory: An introductory overview." *Foundations and Trends in Systems and Control* 1.2-3 (2014): 173-378.

NN controllers

$$\dot{\xi}(t) = (\mathbf{J}_c - \mathbf{R}_c)\frac{\partial \Phi(\xi(t), t)}{\partial \xi} + \mathbf{G}_c{}^\top \mathbf{y}(t)$$

$$\mathbf{u}(t) = \mathbf{G}_c \frac{\partial \Phi(\xi(t), t)}{\partial \xi}$$

- $\mathbf{J}_c$ skew-symmetric
- $\mathbf{R}_c \succeq 0$

- Elements in green are free to be chosen!
- Φ: time-varying Hamiltonian function
  - Continuously differentiable
  - Radially unbounded

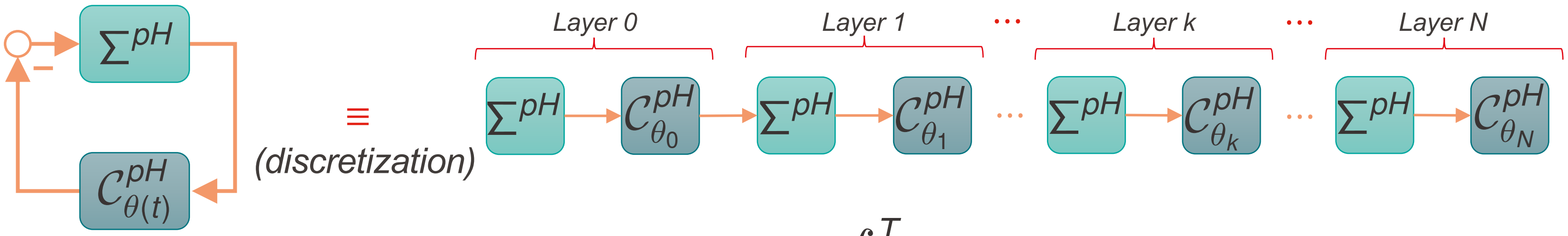
Use a (deep) NN for parametrizing Φ

$$\Phi(\xi(t), t) = \Phi(\xi(t), \theta_\phi(t))$$

- pH NN controllers $\subset$ NeuralODEs[1]

[1] R.T. Chen, Y. Rubanova, J. Bettencourt, and D.K. Duvenaud, “Neural ordinary differential equations”, *Advances in neural information processing systems*, vol *31*, 2018.

NN controllers



- Choose θ minimizing the cost: $\mathcal{L} = \int_0^T \ell(\mathbf{x}(t), \mathbf{u}(t), \boldsymbol{\theta}(t))\, dt$
- Analogous to NN training!
  - Number of layers: N
  - Discretization step size: $h = T/N$

$$\mathcal{L}_{DT} = \sum_{k=0}^{N} \ell_{DT}(\mathbf{x}_k, \mathbf{u}_k, \boldsymbol{\theta}_k)$$

- Closed-loop stability → For an arbitrary $T > 0$, i.e. arbitrary network depth

  → When θ^* is a local minima

  … but also before convergence! → I.e. for θ such that $\nabla_\theta \mathcal{L} \neq 0$

Non-vanishing gradients during training

Theorem[1]**.** If no dissipation in the loop, then

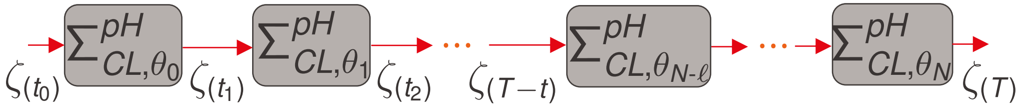
$$\left\| \frac{\partial \zeta(T)}{\partial \zeta(T-t)} \right\| \geq 1$$

$$\zeta = \begin{bmatrix} \text{system state} \\ \text{controller state} \end{bmatrix}$$

- Why?

$$\frac{\partial \zeta(T)}{\partial \zeta(T-t)} \text{ is symplectic, i.e. } \boldsymbol{\Psi} = \left(\frac{\partial \zeta(T)}{\partial \zeta(T-t)} \right)^\top \boldsymbol{\Psi} \frac{\partial \zeta(T)}{\partial \zeta(T-t)}$$

Interconnection matrix of the closed-loop system



[1] L. Furieri, C. Galimberti, M. Zakwan and G. Ferrari Trecate, “Distributed neural network control with dependability guarantees: a compositional port-Hamiltonian approach”, *arXiv preprint arXiv:2112.09046*, 2021.

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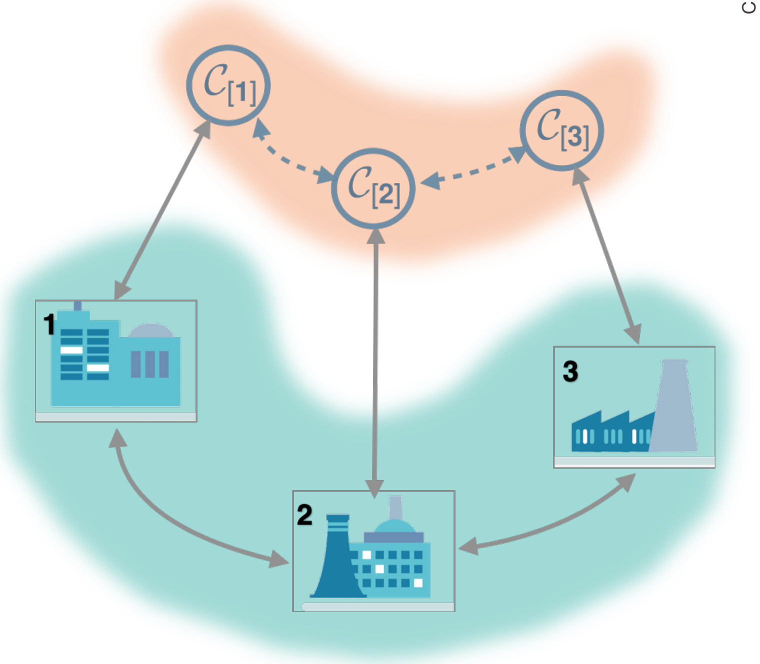
Distributed pH NN controllers

- : Network of pH systems
  - power-preserving interconnections
-  : Network of pH controllers?
  - Local energy: $\Phi_i(\xi_i, \text{neighbors}(\xi_i))$
    e.g. $\Phi_1(\xi_1, \xi_2)$
  - pH if $\dot{\xi}_i$ depends on $\frac{\partial \Phi_i}{\partial \xi_i}$ only?

↓

NO



Distributed pH NN controllers

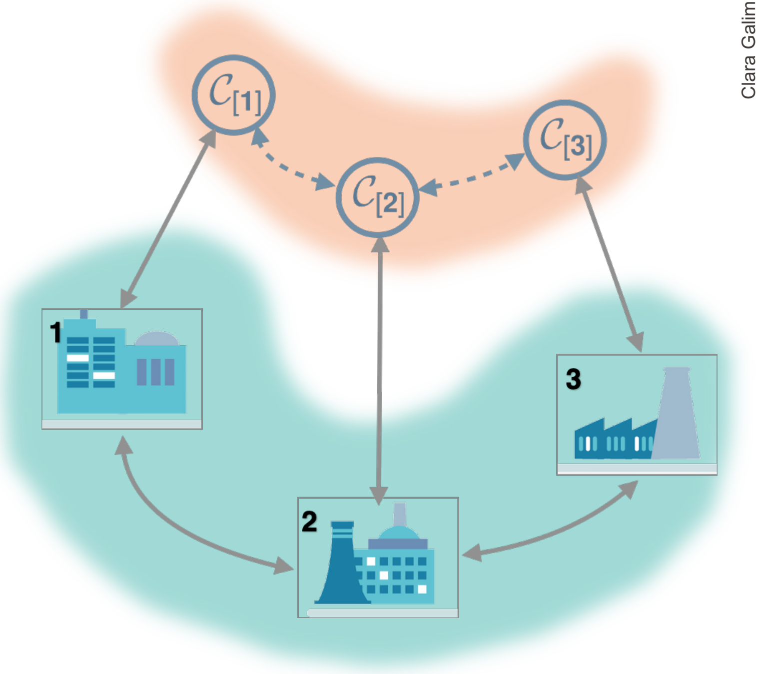
- Solution:
  - Define a global energy $\Phi = \sum \Phi_i$
  - Make $\dot{\xi}_i$ depend on $\frac{\partial \Phi}{\partial \xi_i}$

- Communication requirements? (set $i = 1$)

$$\frac{\partial \Phi}{\partial \xi_1} = \frac{\partial \Phi_1(\xi_1, \xi_2)}{\partial \xi_1} + \frac{\partial \Phi_2(\xi_1, \xi_2, \xi_3)}{\partial \xi_1}$$

ξ_3 is needed in location 1!

The given network is not enough!



- **Problem:** on which controller states should Φ_i depend upon?
  - for the prescribed communication network
  - while guaranteeing to be pH

Distributed pH NN controllers

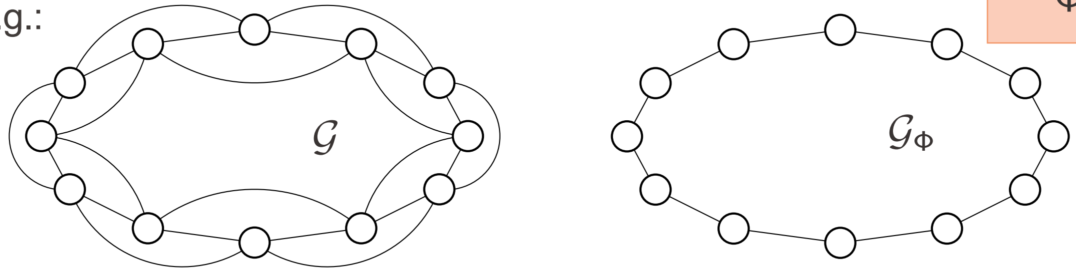
- $\mathcal{G}$ → prescribed communication network
- $\mathcal{G}_\Phi$ → which local energy depends upon which state

Theorem[1]**.** Let $\mathcal{G}_\phi$ be the communication graph describing the state dependencies of the local energies. Then, the NN control policies are distributed according to a prescribed interconnection network $\mathcal{G}$ if

$$\mathcal{G}_\phi^2 \subseteq \mathcal{G}\,.$$

Trivial solution:
Local energies only depend on local states, i.e.
$\Phi_i(\xi_i)\,,\quad \forall i = 1, \dots, M$

E.g.:



- Extension including input-output coupling between controllers can be found in [1]

[1] L. Furieri, C. Galimberti, M. Zakwan and G. Ferrari Trecate, “Distributed neural network control with dependability guarantees: a compositional port-Hamiltonian approach”, *arXiv preprint arXiv:2112.09046*, 2021.

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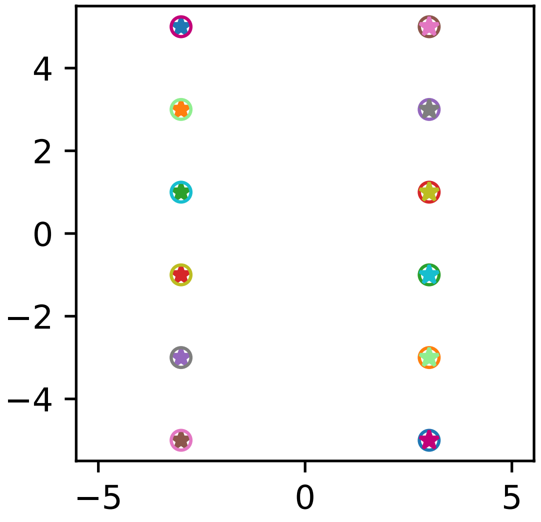
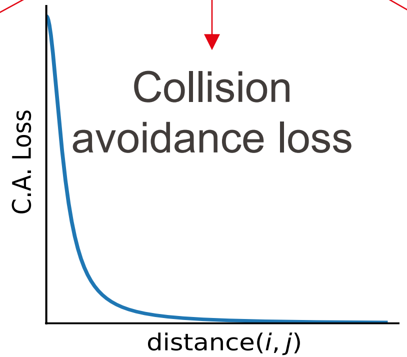
Navigation task using pH NN distributed controllers

- 12 mobile robots in *xy*-plane
  - Modelled by linear point masses
  - **Objective:** navigation ($\star \rightarrow \circ$) within a given time T **+** collision avoidance
  - Prestabilized dynamics around $\bar{\mathbf{x}}$ ($\circ$)
  - **Finite horizon stage cost:** $\ell_Q + \ell_{CA} + \ell_R$



Quadratic loss penalizing:
- Distance to target point
- Non zero velocity
- Input magnitude



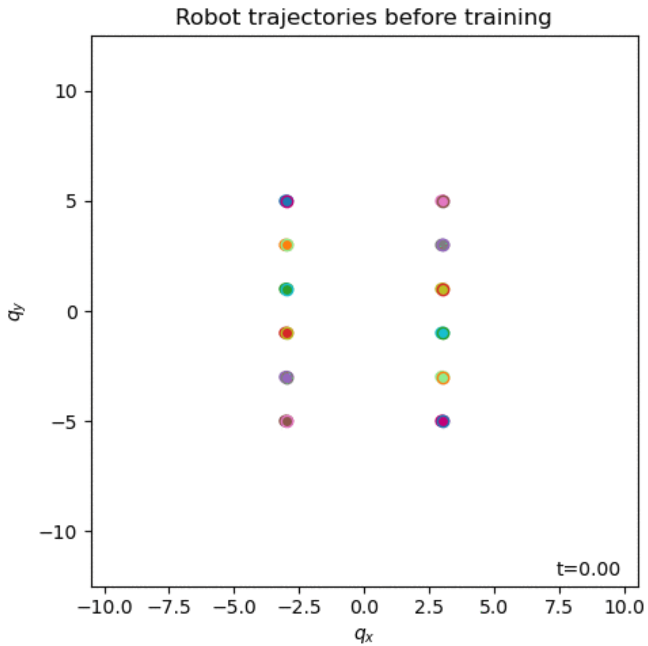
Collision avoidance loss

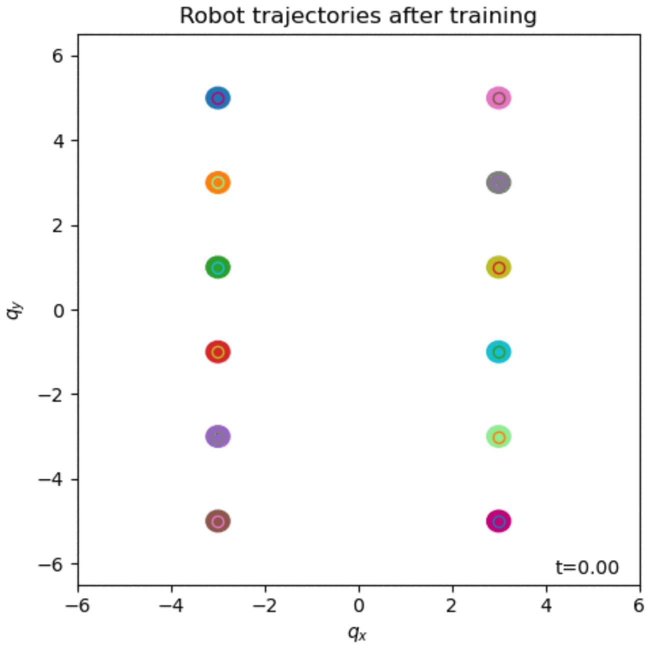
Regularization loss for pH-DNNs

Smoothing parameters across layers

Navigation task using pH NN distributed controllers

Clara Galimberti





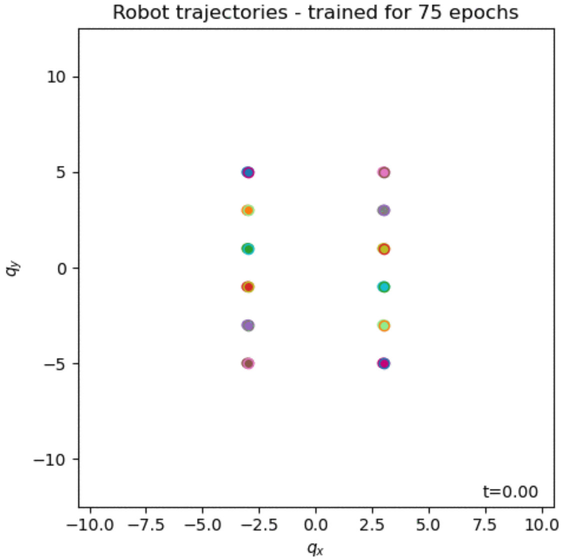
Zero collisions after training

Stability is guaranteed *by design*

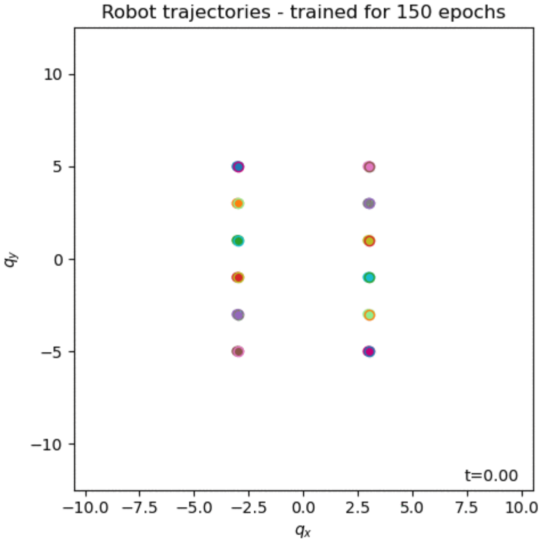
Gifs can be found in our GitHub repository:
https://github.com/DecodEPFL/DeepDisCoPH

Navigation task using pH NN distributed controllers

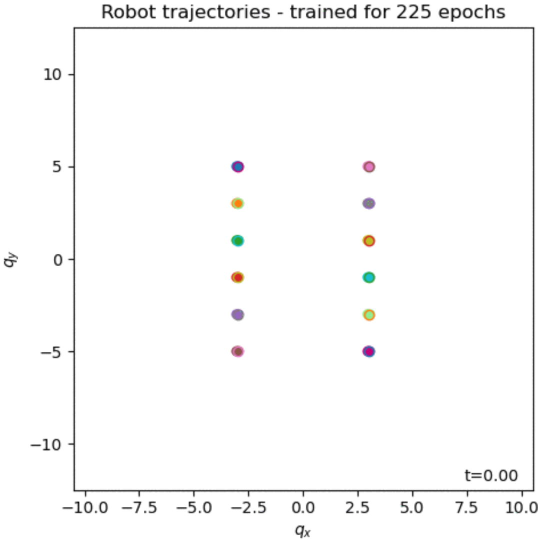
- Early stopping of the training:



25% of the training



50% of the training



75% of the training

Stability is always guaranteed

Gifs can be found in our GitHub repository:
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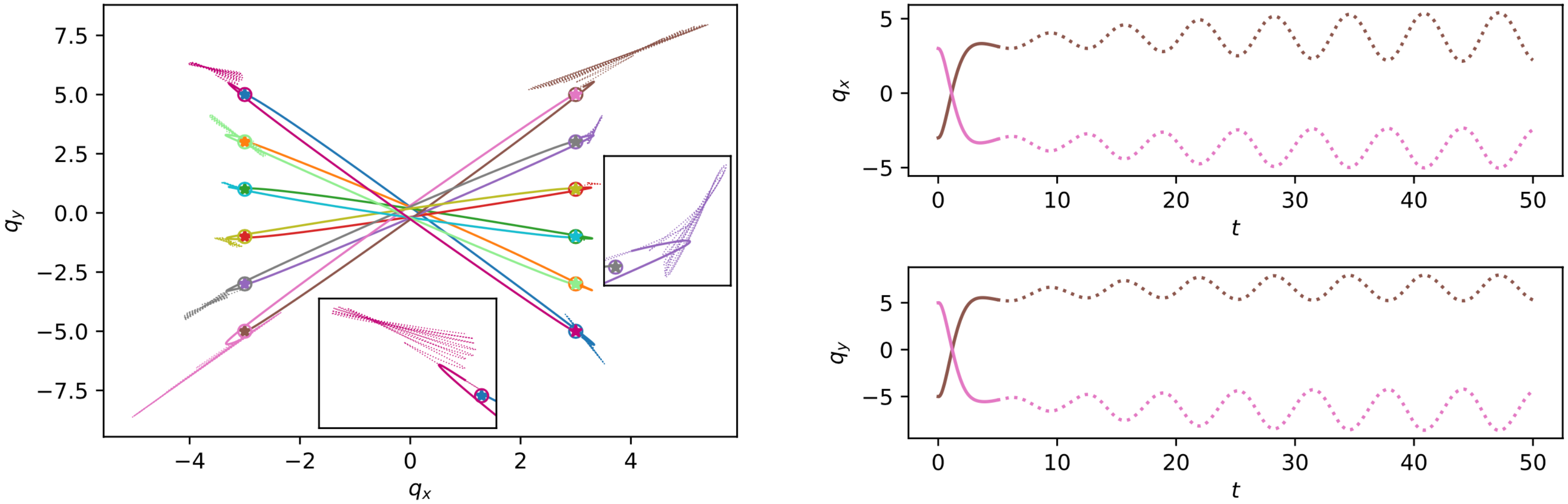
Navigation task using pH NN distributed controllers

- Replacing neural port-Hamiltonian controllers with MLP networks

Results after training → even when not considering collision avoidance



No stability guarantees

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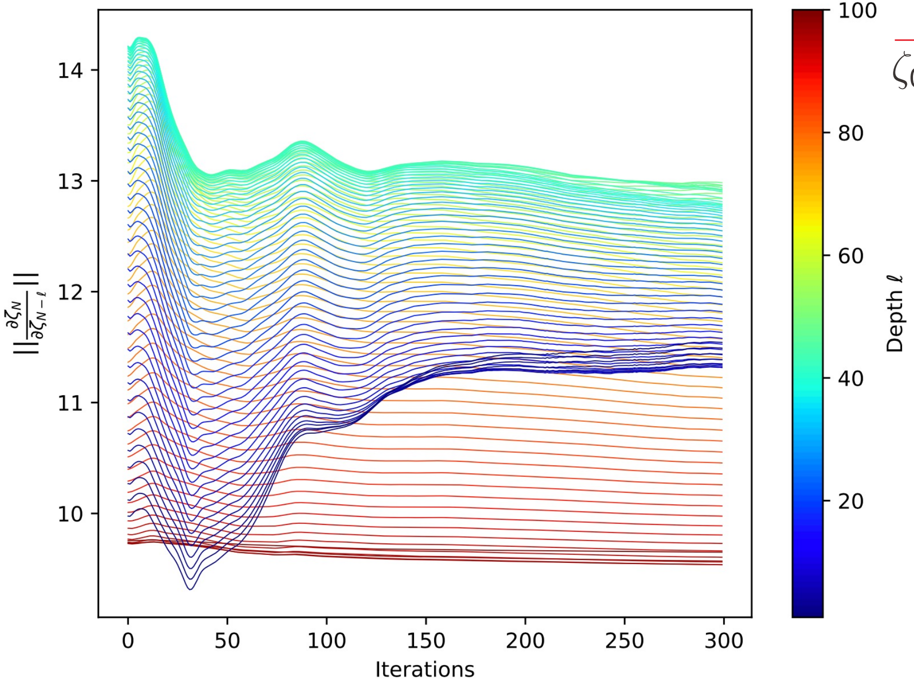
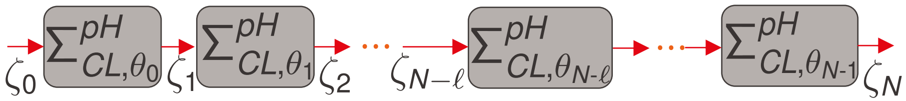
Navigation task using pH NN distributed controllers

- Gradients during training

Backward sensitivity matrix norm





$$\left\|\frac{\partial \zeta_N}{\partial \zeta_{N-\ell}}\right\| \geq 1$$

validates the absence of vanishing gradients during training

Conclusions

- pH NN control policies:
  - Stability of the closed loop *by-design,* i.e. for arbitrary parameters
  - Non-vanishing gradients during training
  - Distributed implementations complying with pH structure

- Next steps?
  - Stable NN controllers by design beyond pH? $\longrightarrow$ [2]
  - Going data-driven: How to incorporate uncertainties in the system modelling?

[2] L. Furieri, C. Galimberti and G. Ferrari Trecate, “Neural System Level Synthesis: Learning over All Stabilizing Policies for Nonlinear Systems”, *arXiv preprint arXiv: 2203.11812*, 2022.

Thank you for your attention!